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A methodological approach to analyze vulnerability of interdependent infrastructures

Min Ouyang [a,b], Liu Hong [a,*], Zi-Jun Mao [a], Ming-Hui Yu [a,b], Fei Qi [a,b]

[a] *Institute of Systems Engineering, Huazhong University of Science and Technology, Wuhan 430074, PR China*
[b] *Key Lab. for Image Processing and Intelligent control, Huazhong University of Science and Technology, Wuhan 430074, PR China*

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ABSTRACT

The infrastructures are interconnected and interdependent on multiple levels, the failure of one infrastructure can result in the disruption of other infrastructures, which can cause severe economic disruption and loss of life or failure of services. A methodological approach to analyze vulnerability of interdependent infrastructures has been introduced in this paper, two types of vulnerability are studied: structural vulnerability and functional vulnerability. Infrastructure topologies are only used for analysis on structural vulnerability while operating regimes of different infrastructures are further considered to analyze functional vulnerability. For these two types of vulnerability, interdependent effects are mainly studied and the effects of interdependence strength between infrastructures have also been analyzed. The analysis on structural vulnerability will be helpful to design or improve the infrastructures in the long run while the discussion on functional vulnerability will be useful to protect them in the short term. The methodology introduced in this paper will be advantageous to comprehensively analyze the vulnerability of interdependent infrastructures and protect them more efficiently.

1. Introduction

The infrastructures on which our society depends are interconnected and interdependent on multiple levels. For example, natural gas may fuel critical gas-fired generators in the electric power system, while at the same time electric power may be used to operate critical systems needed for delivering gas [1]. The failure of one infrastructure can result in the disruption of other infrastructures, which can cause severe economic disruption and loss of life or failure of services which impede public health and well-being. The major power blackout on August 14, 2003, lasted up to 4 days in various parts of the eastern USA, caused traffic's congestion and affected many other critical infrastructures, the estimated direct costs were between $4 billion and $10 billion [2].

In the past few years, many researchers have concentrated on the modeling and analysis of interdependent infrastructures. Related work tends to fall into two broad categories, structural analysis and functional analysis. Based on complex network theories [3,4], many researchers have focused on structural vulnerability of different types of single infrastructure [5–7] or interdependent infrastructures [8–10]. Their responses to attacks are measured by their global and local connectivity, their vertex degree distributions, and their redundancy ratios and so on. However, considering infrastructure topologies alone are not enough to reflect real vulnerability of those different types of infrastructures, so many other researchers have studied functional vulnerability by further taking their operation regimes into consideration. An interdependent layer network model has been formulated by Earl et al. In their works, definitions of five types of infrastructure interdependencies

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[*] Corresponding author. Tel.: +86 02787540210.
E-mail address: newtorrent@sina.com.cn (L. Hong).

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have been presented and incorporated into a network flows mathematical representation [11]. This model has been used to simulate the September 11 attacks on the WTC. A graph Model of Critical Infrastructure has been used for the analysis of infrastructure interdependencies which could take into account not only abstract interdependencies but also selected properties of infrastructure types such as buffering of resources [12–15]. This model tries to use one universal model to describe several kinds of infrastructures, which seem to be difficult, because operating regimes for various infrastructures are varied and simulation steps for various infrastructures should be also different. Recently, Rosato et al. have used the DC power flow model and a model of the TCP/IP basic features to analyze the functional vulnerability of interdependent Italian electrical transmission network and the backbone of the internet network [16]. Their studies have opened the way to the use of simulations of the behavior of interdependent infrastructures based on the use of phenomenological master equations [17].

In this paper, we will introduce a methodological approach to comprehensively analyze the vulnerability of interdependent infrastructures, two types of vulnerability will be considered: structural vulnerability and functional vulnerability. Relationships between these two types of vulnerability will be also investigated. The interdependent infrastructure topologies will be generated based on geographical proximity. Vulnerability is related to attacks and can be described as the decrease of efficiency after attack. For structural vulnerability, the infrastructures topologies are the only information and the average reciprocal shortest path lengths of infrastructure networks are used to measure structural efficiency. For functional vulnerability, operating regimes of different infrastructures are further considered and the functionality levels of these infrastructures are considered as functional efficiency. When infrastructures are subjected to attacks, infrastructure efficiencies decrease and their vulnerability can be analyzed. Electrical network and gas pipeline network are mainly discussed in this paper. The gas pipeline consists of metering stations, valves, compressors and gas storages, all these components depend on power supply. Some electric power is provided from turbines driven with gas from the pipelines [12]. For different infrastructures, various models will be used to describe their operating regimes.

The rest of this paper is organized as follows: Section 2 introduces the methodological approach to comprehensively analyze the vulnerability of interdependent infrastructures. The analysis on structural and functional vulnerability, including the operating regimes for different infrastructures will be also introduced in this section. In Section 3, based on the methodological approach, artificial interdependent infrastructures will be generated and simulations are then performed to analyze their vulnerability, and effects of interdependence strength between these two infrastructures are also discussed. Finally, what we found in our paper are summarized in Section 4, and future directions are proposed.

2. A methodological approach

In this section, a methodological approach to comprehensively analyze the vulnerability of interdependent infrastructures will be introduced, two types of vulnerability are considered: structural vulnerability and functional vulnerability. For structural vulnerability, infrastructures topologies are the only information while operating regimes of different infrastructures are further taken into consideration to analyze functional vulnerability. The vulnerability analysis process of interdependent infrastructures can be seen in Fig. 1.

From the figure, the first step is to extract the topology of each infrastructure, i.e. what are described by nodes and what are modeled by links. When infrastructure topologies have been extracted, their operating regimes can be further considered

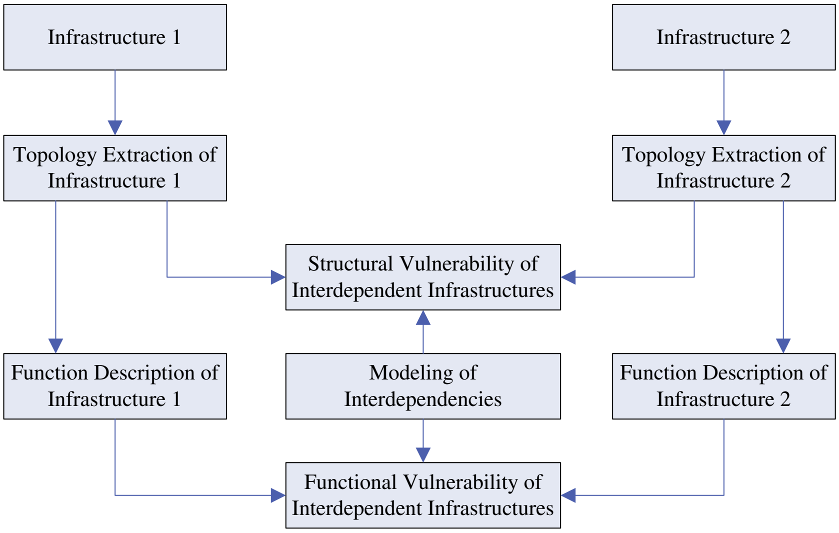

Fig. 1. The vulnerability analysis process of interdependent infrastructures.

for analysis on functional vulnerability. However, no matter it is the analysis on structural vulnerability or functional vulnerability, the most important thing is to model interdependences between two infrastructures. There are several types of interdependences between infrastructures. Different scholars have different view on the classification [18,19]. Recently, Earl et al. have concluded five types of interrelationship between infrastructure systems [11], namely, input dependence, mutual dependence, shared dependence, exclusive-or dependence and co-located dependence. In this paper, the mutual dependence and co-located dependence will be considered. Electrical network and gas pipeline network will be mainly discussed in this paper, where some gas nodes depend on the nearest power nodes to work while some power generators will be driven by gas from the nearest gas nodes. The concrete interdependences between these two infrastructures will be further introduced in the following simulation.

According to Fig. 1, in the following context, we will first introduce topologies extraction for each infrastructure, and then introduce the analysis on structural and functional vulnerability.

2.1. Topology extraction

As stated above, to comprehensively analyze the vulnerability of interdependent infrastructures, the fist step should be to extract the topology of each infrastructure, i.e. what are described by nodes and what are modeled by links. Each infrastructure can be described as networks in different ways. For example, for road network, intersections can be described as nodes while road sections can be modeled by links, but we can also use nodes represent road sections while links mimic their relationship, i.e. if two road sections have an intersection, one link will connect the corresponding two nodes. It is also the same for gas pipeline system.

In this paper, we will mainly consider interdependent electrical network and gas pipeline network. For electrical network, generated power originates from a small number of power generators, and the power is delivered to a large number of transformers, which serve the low voltage distribution network, potentially through several intermediate sub-distribution networks [20,21]. In the following discussion, nodes are used to represent generators or transformers or substation and links mimic electrical wires between them. For gas pipeline network, it can be divided into three main parts: the gathering system, the transportation system, and finally the distribution system. The gathering system consists of low pressure, low diameter pipelines that transport raw natural gas from the wellhead to the processing plant. Transmission pipelines move gas in large quantities over long distances with few or no major supplies or off-takes between the end points of the pipeline. The gas pipeline segments will be described as nodes. The branches will be the edges [22].

As it is difficult to acquire all related data about real interdependent infrastructures, artificial infrastructure topologies will be generated and used for vulnerability analysis. It will be further introduced in following simulation analysis.

2.2. Analysis on structural vulnerability

Vulnerability is related to attacks and can be described as the decrease of system efficiency after attack. To analyze structural vulnerability, infrastructure topologies are only considered and the most important thing is to determine what are used to describe structural efficiency. In Refs. [23,24], there are many definitions on structural efficiency, such as average shortest distance, network diameter, and cluster efficiency, but they all have some limitation. Usually, the average reciprocal shortest path lengths of networks are used to measure the structural efficiency and it is generally accepted [24]. It will be also used in this paper for simulation analysis.

The topology is represented as a graph $G = \{V, E\}$ with N nodes, $V = \{v_i\}$ is the set of vertices and E is the set of edges, denote $d(v_i, v_j)$ by the shortest path lengths connecting two nodes in the network, then the structural efficiency $X(G)$ of one infrastructure can be defined as follow [25–27]:

$$X(G) = \frac{1}{N_r N_l} \sum_{i \in G_r, j \in G_l} \frac{1}{d_{ij}}, \tag{1}$$

where N_r is the number of resource nodes (such as generator nodes in the power network and pump nodes in the gas network) and N_l is the number of load nodes (nodes offer services to other systems). When two nodes are not connected at all, or become disconnected due to attacks, their shortest path length $d(v_i, v_j)$ becomes infinite, and then $1/d(v_i, v_j)$ is zero. If $X(G)$ is large, it is indicated that the network is well connected and has high efficiency. In following simulation, $X(G)$ will be normalized in the range $[0, 1]$.

In addition, another important thing is to model the structural interdependence. For analysis on structural vulnerability, when one power node is attacked, all gas nodes supplied by this power node will be deleted. Similarly, when a gas node is attacked, the corresponding gas-based power generators will be also removed. This will change network topologies. The structural efficiency can be calculated and structural vulnerability can be further analyzed.

2.3. Analysis on functional vulnerability

In above subsection, we have mainly introduced the analysis on structural vulnerability. To analyze the functional vulnerability, operating regimes of different infrastructures should be further considered. In this subsection, the analysis on functional vulnerability will be introduced. Based on above topologies extraction, an electric power model and a gas pipeline model will be firstly given, and then the functional interdependence will be further modeled.

2.3.1. Electric power model

Recently, Dobson et al. have given an overview of a complex system approach to large blackout of electric power transmission systems caused by cascading failures [21], many blackout models can be seen in that paper. In this paper, we use a DC power flow model to describe the transport of electrical flow in the network. As stated before, nodes are used to represent generators or transformers or substation and there are two types of nodes in the electrical network, namely, generator nodes which produce power and load nodes which offer power to other systems. The network nodes are characterized by the input power, P_i, which is positive for generators and negative for loads, and the maximum power that a generator can supply is $P_i^{\max}$. Each network transmission line connects two nodes, i and j, and is characterized by the power flow through the line, F_{ij}, and the maximum power flow that it can carry, $F_{ij}^{\max}$, and the reactance of the line $x_{ij.}$ The DC power flow equations provide a linear relationship between the power flowing through the lines and power input into the nodes. This equation can be written as $\boldsymbol{F} = \boldsymbol{AP}$, where $\boldsymbol{F}$ is a vector whose N_1 components are the power flows F_{ij} through the lines, $\boldsymbol{P}$ is a vector whose $N_N - 1$ components are the input power P_i of each node, except the reference node P_0, and $\boldsymbol{A}$ is a constant matrix, whose elements can be calculated in terms of the reactance of the lines. More detail on this equation can be seen in Ref. [28].

When a node is attacked, it is necessary to redispatch the injected powers to satisfy the system constraints. The injected powers include both generators and loads but generation redispatch is much preferred to load shedding. The redispatch is formulated in a conventional way as an optimization to minimize the change in generation or load shed subject to the system constraints. The optimization minimizes the cost function:

$$\sum_{\text{generators}} |p_i - P_i| + \sum_{\text{loads}} W|p_i - P_i|. \tag{2}$$

In this model, we assume all generators run at the same cost and all loads have the same priority to be served. However, we set up a high price for load shed by setting $W = 100$. The minimization of cost function is done with the following constraints: (1) Total power generated and consumed must balance: $\sum_{i \in \text{generators} \cup \text{loads}} (p_i - P_i) = 0$; (2) Power flow through the line is limited: $|f_{ij}| \leqslant F_{ij}^{\max}$; (3) Limits on the generator power: $0 \leqslant p_i \leqslant P_i^{\max}$, $i \in$ generators; (4) Load shedding should be positive and less than the total load: $P_i \leqslant p_i \leqslant 0$, $i \in$ loads. This optimization problem can be changed into a standard linear programming (LP) problem. Then we can find the new input power for each node, and the power supply can be calculated after attack. In this paper, we use the decrease of power supply to reflect the functional vulnerability after attacks.

2.3.2. Gas pipeline model

Natural gas pipeline systems transport natural gas from sources to users through a system of interconnected pipeline segments. The difference in pressure at different points of the pipeline is the force driving the gas through the pipes [12,22]. As stated before, the gas pipeline segments will be described as nodes and the branches will be the edges. The modeling of pipeline has long been a subject of interest as made evident by the early modeling efforts by Kralik et al. [29]. Some related studies can be found in Refs. [12,30,31]. In this paper, we will use the dynamic approach of the model in the paper [12], and small modifications will be made. Every pipe segment is represented as a node v_i. If two segments have an intersection, an edge will be added to connect the two nodes. Each node has a buffer of volume V_i, and a pressure limitation $P_{\max}(v,i)$. The relation between pressure and volume for gas is given by the ideal gas law $PV = nRT$, where n is the number of moles, R is the universal gas constant, and T is the temperature. As an approximation, the temperature is assumed to remain constant in the pipeline, and normalizing the relations such that $RT = 1$, and then $PV = N$; where N is the amount of some arbitrary unit of gas. The difference in pressure between different segments is the force driving gas from one node to another. Given the pressure P_i and P_j of two connected nodes v_i and v_j, the pressure difference is denoted by ΔP_{ij}, so the amount of gas flow from nodes v_i to v_j at the time t can be defined as follows:

$$R_{i\to j}(\Delta P_{ij}(t), t) = \begin{cases} 0 & \text{if} \quad \Delta P_{ij} < \frac{f_{\max}}{10}, \\ \frac{f_{\max}}{1+e^{2.5-10*\delta P_{ij}(t))/f_{\max}}} & \text{if} \quad \Delta P_{ij} \geqslant \frac{f_{\max}}{10}, \end{cases} \tag{3}$$

where ΔP_{ij} is the pressure difference at the time t, and $f_{\max}$ is the possible maximum capacity of the flow from nodes v_i to v_j at the time t. If $p_i < p_j$, the maximum possible capacity of the flow from nodes v_i to v_j at the time t will be $V_j^*(p_j - p_i)/2$. Let $r_i(t)$ be the amount of gas resource produced in the node v_i at time t. So the amount of gas available in a pipe segment at time $t + 1$ is given by:

$$N_i(t+1) = r_i(t) + N_i(t) + \sum_{j\in V} R_{j\to i}(\Delta P_{ji}(t), t) - \sum_{j\in V} R_{i\to j}(\Delta P_{ij}(t), t). \tag{4}$$

By computing the formula (4), the gas flow through the network over time can be observed. In addition, the order of the considered edges influences the result, so the sequence of edges is altered from time step to time step, and then the behavior of the intersection is correct on average. Even more, every pipeline components has a pressure limitation. Thus it must be possible to diminish or eliminate $r_i(t)$ and the valves should be closed when incoming pressure is large. All this is implemented in our paper. The goal of a gas pipe is to deliver gas to customers. In our model the customers are located as leaf nodes (their degrees equal one) in the gas network. We choose to measure the functionality of gas network in terms of the fraction of leaf nodes receiving sufficient amount of gas to cover their needs.

2.3.3. Modeling Interdependencies

Unlike structural interdependence, it is more complicated to model functional interdependence. When some power nodes are attacked, the power injections will be redispatched so that some unattacked power nodes cannot supply sufficient power for the corresponding gas nodes and further the gas nodes will be closed. Similarly, when some gas nodes cannot supply sufficient gas for the power generator, the generator will have to close. In following simulation, we consider simple interdependence. If the power supply of one power load node is less than *ps* fraction of its normal power, the gas pipelines that are served by that power node will be closed. This can make the customers located as leaf nodes cannot receive gas when the gas stored in the nodes are depleted. When the gas node for power production has none gas, the corresponding gas-fired generator will be removed and the power injections will be redispatched again so that some power nodes will be not able to dispatch sufficient power, further some gas nodes will be closed and then the efficiencies of both the two networks will decrease.

In addition, in this paper, no matter it is the analysis on structural vulnerability or functional vulnerability, we will mainly consider their interdependent effects, and a parameter called “interdependent effect” will be introduced. This parameter is defined as the absolute difference between the independent and interdependent efficiency and the difference is normalized by the maximum independent efficiency attained at any removal fraction

$$\text{Interdependent effect} = \frac{|\text{Interdependent Efficiency} - \text{Independent Efficiency}|}{\max(\text{Independent Efficiency})}. \tag{5}$$

For the independent scenario, the efficiency can be calculated for any given removal fraction. But for the interdependent case, a fixed fraction of nodes will be removed for both two networks, and then the interdependent efficiency is calculated for each network. More details will be described and the parameter “Interdependent Effect” will be further studied in the following simulation.

In this section, we have introduced a methodological approach to comprehensively analyze the vulnerability of interdependent infrastructures. The topologies extraction and analysis on structural and functional vulnerability have been described. In the next section, artificial interdependent infrastructures will be generated, and then these two types of vulnerability will be further analyzed. Their relationship will be also discussed.

3. Simulation analysis

In this section, according to above methodological approach, the vulnerability of interdependent infrastructures will be analyzed. As related data about real infrastructures is difficult to acquire, artificial interdependent infrastructures will be generated and the vulnerability will be further discussed by means of simulation.

3.1. Topologies generation

In this paper, the proximal topology generator will be used to construct infrastructure topology, this generator is introduced by Michael [32]. The networks are created based on the spatial proximity of nodes during network growth. The graph is initially seeded with a root node and assigned a random location. At each time step, a new node is added to the graph and a new undirected edge is linked to existing nodes in the graph based on minimum Euclidean distance. This algorithm has been extended to a more general algorithm. Xu et al. have studied the network generation on Euclidean space [33], in his algorithm, each new nodes is added and placed in a random location on a Euclidean space while it connects to the exiting node with a probability which is linearly proportional to $k_i^{\beta}(t)d^{a}$, where $k_i(t)$ is the degree of existing nodes at the time t, d is the distance between the new node and existing node, and α and β are modulated parameters. When $\alpha = -\infty$ and β is limited value, each new node will only connect with the nearest existing node, which is the same as above algorithm.

In this paper, we will use above algorithm to generate infrastructure topology and each new node will connect with the nearest existing node. Specifically, for power distribution network, the following procedure is used to generate power network: (1) Initially, the graph is seeded with several independent nodes as power generators. No links among them. (2) At every time step, a new node with a new edge is added to the network. The other end of the edge is connected to existing node in the graph based on minimum Euclidean distance. (3) After the final time step, a sparse random graph is placed on the top of the network.

According to above procedure, the electrical network can be generated. Without losing generality, only two types of nodes will be considered for electrical network: generator nodes and load nodes. For the gas pipeline system, just like the generating procedure of electrical network, the gas network can be also generated, and there are three kinds of nodes in the networks: pump nodes, transmission nodes and load nodes. In addition, in this paper, to describe their interdependence, the mutual dependence and co-located dependence will be considered. Some gas nodes depend on some power load node to supply power and work well while some power generators are driven by gas from the pipelines. To simulate their

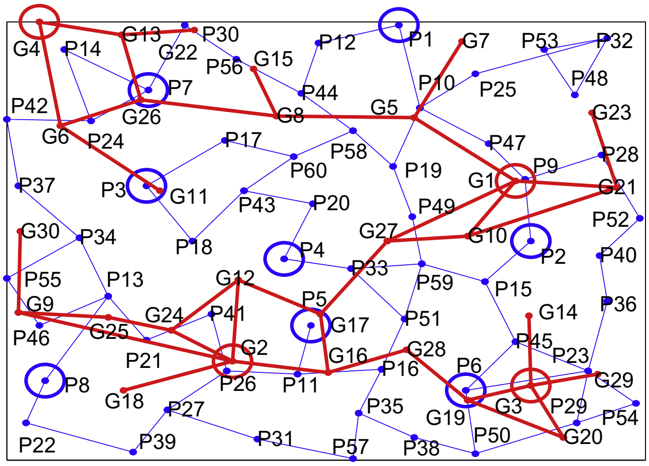

Fig. 2. The electric power network (the blue color, P1–P60) and the gas pipeline network (the red color, G1–G30). The nodes with blue circles (P1–P8) are the generating station, while the nodes with red circles (G1–G4) are the pumps for gas. The blue lines are the electric wires while the red lines are the branching connecting two gas pipeline segments.

interdependence, the two networks are generated in the same area, and some gas nodes will be served by the nearest power load nodes while some generators will be driven by the nearest gas nodes.

Based on above procedures, both networks are generated in the same graph, as can be seen in Fig. 2. The infrastructure topologies generated in the figure have the similar features with the real infrastructures. They both are homogeneous networks. Specifically, the power network has 60 nodes and 75 edges, eight of these nodes are generators nodes[1] (blue circle, P1–P8), others (P9–P60) are load nodes. The average degree of this network is 2.5, it is near to the value for the north American grid (2.78) [6], and the grid in an area of China (2.36) [34]. The gas network has 30 nodes, four of them are pumps (red circle, G1–G4), the leaf nodes whose degrees equal one are used to serve the customers, such as power production. Some gas nodes will depend on their nearest power load nodes to work well, and some leaf nodes in the gas network will be used for power production. The interdependence strength (such as coupling parameters and number of links between two infrastructures) will be further introduced and discussed in the following simulation.

Based on above artificial interdependent infrastructures, the structural vulnerability and functional vulnerability will be further studied in the following subsections, and their relationships and the interdependence strength will be also discussed.

3.2. Structural vulnerability

To analyze the structural vulnerability, we will mainly consider the pure random attacks. Many works have concerned on the structural vulnerability of single network [25–27], a few researchers have studied that of interdependent networks [8]. In this paper, the interdependent effects will be mainly investigated. The simulation process is done as follows: Firstly, fix the fraction of removed power and gas nodes. Secondly, subject both networks to simultaneous attacks, and a fixed fraction of gas or power nodes are removed from both two networks. Different from the simulation process in Ref. [8], in our simulation, if there is none of path from any power generators to a power load node, this power load node will be removed and the corresponding gas node will be also closed due to lack of power supply. Similarly, if there is none of path from any gas pumps to a gas node, the gas node will be removed and the corresponding gas-based generator will be also removed. Finally, calculate the efficiency of remaining networks. The additional removal fraction levels range from 0 to 1 at increments of 1/30. In addition, to reflect the interdependent effect, the case that the two networks are independent is also considered. Fig. 3 shows normalized structural efficiency in dependence of the fraction of removed nodes for different networks. In this simulation, all gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator and this generator is driven by the nearest gas node. The final simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator.

As can be seen in Fig. 3, the gas network has small effect on the power network while the power network has large influence on the gas network. Further, to reflect the effect of their interdependence clearly, the parameter "interdependent effect" in dependence of the fraction of removed nodes can be seen in Fig. 4.

Fig. 4 illustrates the interdependent effect in dependence of the fraction of removed nodes. It can be clearly found that the interdependences have small effect on power network while they have large impact on the gas pipeline network. At their peak, the interdependent effects account for an additional reduction in network functionality with respect to their independent scenarios of approximately 5% and 24%, respectively. These results are obvious because of the special interdependence

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[1] For interpretation of color in Figs. 1–9, the reader is referred to the web version of this article.

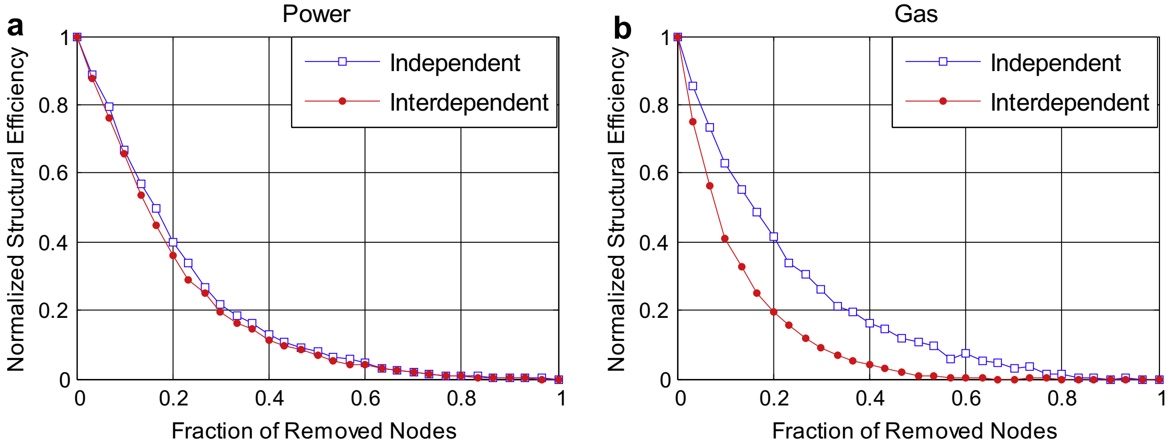

Fig. 3. Normalized structural efficiencies in dependence of the fraction of removed nodes for different networks under random attack. All gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator. (a) Power network; (b) gas network.

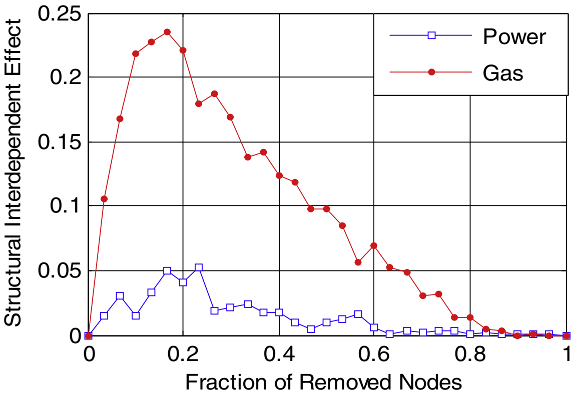

Fig. 4. The parameter "interdependent effect" in dependence of the fraction of removed nodes for different networks under random attack. All gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator.

structure, i.e., many gas nodes need the power load nodes to supply power while there is only one gas-fired power generator. However, it is interesting to find that the interdependent effects for both two networks will reach the maximum when the fraction of removed nodes is round 18%.

Although the results we get above largely depend on interdependent structure, this analyzing approach can be useful to design or improve interdependent infrastructures in the long run. However, to protect our infrastructures in short term, more information are needed and operating regimes of different infrastructure should be further taken into consideration. So in the next subsection, functional vulnerability will be discussed.

3.3. Functional vulnerability

To analyze the functional vulnerability, we should firstly set some functional parameters. In the following simulation, for electrical network, assume all power generators work in normal states and the power injections for all generators and all power load nodes are respectively 4 MPW and 8/13 MPW. The maximum generator power and maximum power flow through lines are assumed to be the products of a tolerant parameter (*tp*) and their normal values. For gas pipeline network, assume the volume and maximum pressure for all gas nodes are, respectively, 1 and 50, the gas demand for all gas load nodes are all 2 and the maximum amount of gas produced for all pumps are all 8. The amount of gas produced for each pump can be adjusted according to operation state and real demand. In addition, for the interdependent structure, just like the consideration in structural vulnerability, all gas nodes depend on the nearest power nodes to supply power and work well while there is only one randomly selected gas-based generator and this generator is driven by the nearest gas node.

As the power flow is fast, but the gas flow is relatively slow, so the simulation step should be different for these two kinds of networks. In our simulation, the simulation step for gas network is assumed to be 1, while the redispatch for power network is done within one simulation step. When the power network is attacked, the simulation process is done as follows: At the time 50, the attack happens. Choose a power node or a gas node randomly and remove that node, redispatch the power injections according to the optimization in Section 2.3.1. At the time 51, the power network will be in the steady state. If the power supply of one power load node is less than *ps* fraction of its normal power, the gas pipelines that are served by that power node will be closed. This maybe makes the customers located as leaf nodes can't receive gas when the gas stored in the nodes are depleted. When the gas node for power production has insufficient gas, the corresponding gas-fired generator will be removed and the power injections will be redispatched again. Some power nodes will be not able to dispatch sufficient power, further some gas nodes will be closed and then the efficiencies of both the two networks will decrease. The simulation runs 100 times for random attacks and 20 times for randomly selected gas-based generator and the averaged functional efficiencies are recorded as the results. When the attack happens on the power or gas network, the functional efficiencies in dependence of the simulation time can be seen in Fig. 5. The tolerant parameter in this simulation is 1.5 and the random attack scenarios (remove a randomly selected node) are mainly discussed.

Compared to the gas flow, the efficiency decrease for power network is instantaneous. So under the same time scale, the efficiency of power network decreases sharply. For random attack on power network (see Fig. 5a), the removal node makes some power nodes are not able to redispatch sufficient power and we can see immediate decrease for the power network at the time 51. As the time pass by, when the gas node for power production has insufficient gas, the corresponding gas-fired generator will be removed, which will further cause the decrease of efficiency of power network. So two steps of functional decrease can be found in the inset of Fig. 5a. Finally, the efficiency of power network decrease 5% and it is 8% for gas network. In addition, due to the amount of gas buffered up in the gas pipeline, on the one hand, there is a time period (about 9 unit time) between the two decrease of power networks in the functionality (see Fig. 5a). On the other hand, for the attack on gas network (see Fig. 5b), it seems to be no immediate effect on the whole system. After about nine iterations, the decays of functionality for gas network are initiated and a few time steps later, the gas-fired power generator is closed and removed, and only one functional decrease happens on the power network. Finally, the efficiency of power network decreases a little (1%).

Just as the same in Section 3.2, to reflect the interdependent effect, the independent scenarios are also considered. For interdependent cases, a fixed fraction of gas nodes and power nodes are removed simultaneous for both two networks. By means of simulation, the functional efficiency in dependence of removal fractions can be seen in Fig. 6. From the figure, just as one single attack scenario, for different removal fractions, the gas network always has small impact on the power network while the power network has large influence on the gas network. Further, to reflect the effect of interdependencies clearly, then we will use the formula (5) to calculate the "interdependent effect". Results show in Fig. 7. At their peak, the functional interdependent effects account for an additional reduction in network functionality with respect to their independent scenarios of approximately 6% and 42%, respectively. To compare with the structural interdependent effect (at their peak, it is approximately 5% for power network and 24% for gas network), the functional interdependent effect on gas network becomes larger, but the phenomenon that the interdependences have much lager effect on gas network than that on power network still hold true, which indicates the special interdependent structure (i.e., many gas nodes need the power load nodes to supply power while there is only one gas-fired power generator) play a key role in this phenomenon.

Although the results we get above still largely depend on interdependent structure and system parameters, this analyzing approach can help the manager to make better decisions on prioritizing scarce resources for mitigation actions, and also be useful to improve the infrastructures in the short term. In addition, it is interesting to find that the optimum removal fraction

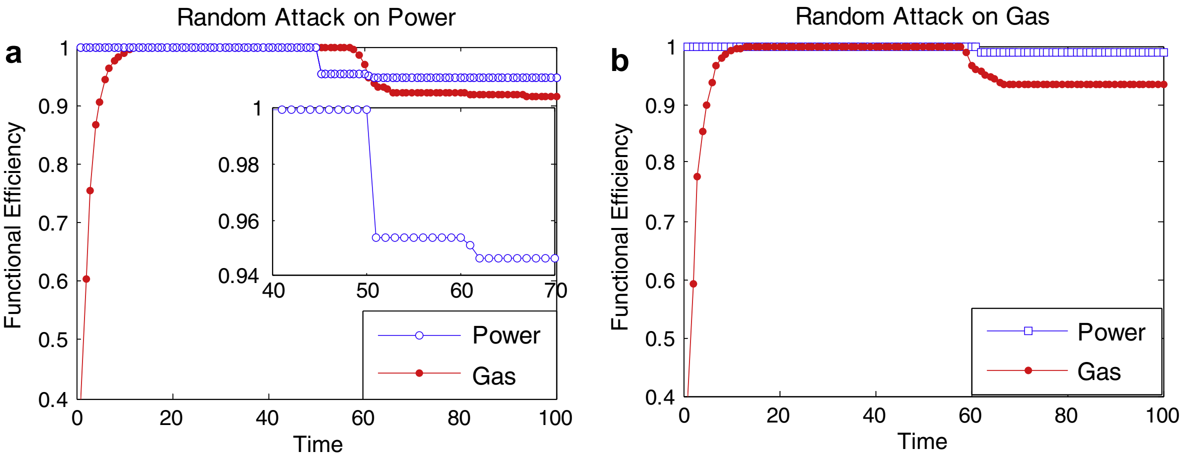

Fig. 5. Functional efficiency in dependence of simulation time when a power or gas node is attacked and removed randomly at the time 50. $tp = 1.5$; $ps = 0.6$. All gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator. (a) Attack happens on power network, the inset is the amplify of the two step failures for power networks. (b) Attack happens on gas network.

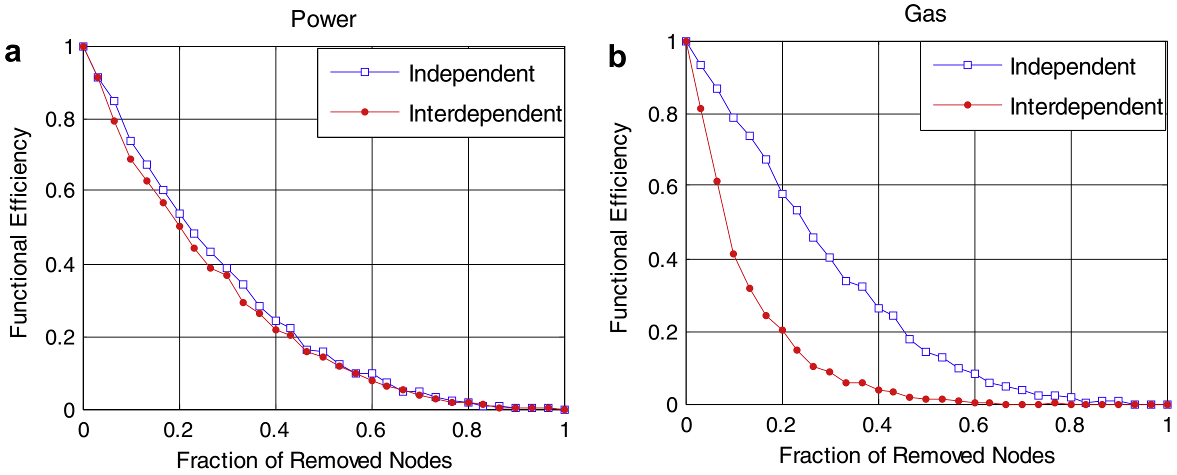

Fig. 6. Normalized functional efficiencies in dependence of removal fraction for different networks under random attack. All gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator. $tp = 1.5$; $ps = 0.6$. (a) Power network; (b) gas network.

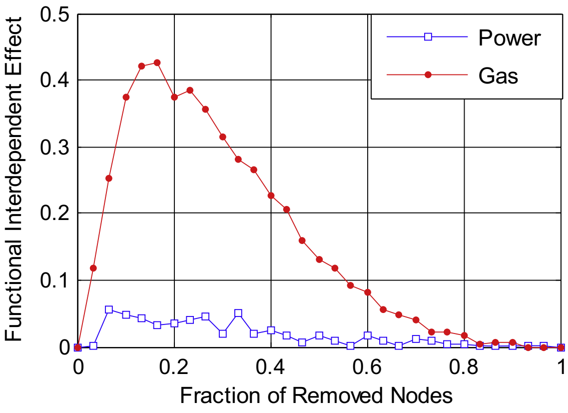

Fig. 7. The functional interdependent effect in dependence of the fraction of removed nodes for different networks under random attack. $tp = 1.5$; $ps = 0.6$. All gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator.

causing the largest functional interdependent effect is the same as that for structural interdependent effect, they are both round 18%. Whether this phenomenon and optimum removal fraction will be affected by system parameters? This will be discussed in the next section.

3.4. Effect of interdependence strength

The structural vulnerability and functional vulnerability have been studied respectively in above two subsections. No matter it is the analysis on structural vulnerability or functional vulnerability, they are both found that the gas network has small impact on the power network, while the power network has large impact on the gas network. However, the functional vulnerability is largely influenced by interdependence strength, which can be changed through two different approaches. The first approach is to change the coupling parameters between the two infrastructures, such as the power strength which can ensure the corresponding gas node operating well, gas amount which can make sure the normal operation of the corresponding gas-fired generator. The second approach is to change the number of links between two infrastructures. In this section, we will discuss the effect of interdependence strength, and relationships between structural vulnerability and functional vulnerability will be also analyzed and discussed.

For the first approach, we will mainly consider the change of coupling parameters mainly caused by the parameter *ps*. By means of simulation, the interdependent effects for different parameter *ps* have been simulated, results show in Fig. 8. To analyze relationships between structural interdependent effect and functional interdependent effect, the structural interdependent effect is also shown in that figure. As can be seen in the figure, on the one hand, it is interesting to find that if the value of parameter is less than 1, no matter it is the structural interdependent effect or functional interdependent effect, the interdependent effects reach the maximum when the fraction of removed nodes is round 18% for both two networks. This optimum removal fraction gives us suggestion to design the best interdependency-based attack strategy that can cause the largest collapse with the least cost. However, to cause the largest interdependent effect, whether the result that the optimum removal fraction for structural interdependent effect is just that for functional interdependent effect still holds true for real interdependent infrastructures and for different types of interdependent network structures? This question is outside the scope of this paper and will be further discussed in future. On the other hand, for different parameter settings, just as the structural interdependent effect on power network and gas network, the functional interdependent effects on power network are always much smaller than those on gas network. In addition, the interdependent effect becomes larger but changes slowly with the increase of parameter *ps*, which indicates the interdependent structure has large impact on interdependent effect and although the coupling parameter are also important, they have relatively less impact. In addition, when the value of parameter *ps* equals 1, if the power of a power supply node is a little less than its normal power, the corresponding gas node will be removed. So the interdependent effect suddenly becomes larger and it is 14% for power network and more than 80% for gas network, but the phenomenon that the interdependent effect on power network is much larger than that on gas network still holds true.

All above results are based on the assumption that all gas nodes depend on their nearest power load nodes to work while there is only one power generator driven by gas, so the structural interdependent effect of gas on power network is relatively weak. Here, we will discuss the second approach and consider the change of the number of links between two infrastructures. In the simulation, all gas nodes still depend on their nearest power load nodes to work while some power generators will be selected randomly and they will be driven by their nearest gas leaf nodes. Define parameter *ngbp* as the number of gas-based power generators. Larger *ngbp* corresponds to stronger correlation for gas network on electric network. By means of simulation, for different value of the parameter *ngbp*, the structural and functional interdependent effect of gas on power network in dependence of fraction of removed nodes can be seen in Fig. 9. Fig. 9a shows the structural interdependent effect of gas network on power network. With the increase of parameter *ngbp*, the structural interdependent effect of gas on power network becomes larger. This is an obvious result, but it is interesting to find that interdependent effect reaches the maximum when the fraction of removed nodes is 18%. Fig. 9b shows the functional interdependent effect of gas network on power network. With the increase of parameter *ngbp*, the functional interdependent effect on gas on power also becomes larger. Unlike the structural interdependent effect, the maximum interdependent effects are reached not for fixed fraction of removed nodes 18%, but it is still round 18%. This could be due to simulations error, although we cannot sure whether there is still a fixed optimum removal fraction for functional interdependent effect, we can sure the optimum removal fraction is round 18%, this is also found in above subsection for different parameters settings for only one gas-based power generators. This approximate optimum removal fraction gives us suggestion to design effective interdependency-based attack strategy that can cause the largest collapse with the least cost, and it can also help the manager to design or protect our interdependent infrastructures.

In this section, according to the methodological approach introduced in Section 2, artificial interdependent infrastructures have been generated and then the vulnerability has been analyzed by means of simulation. The effect of interdependence strength has been also discussed. This interdependence strength is changed mainly through the change of coupling param-

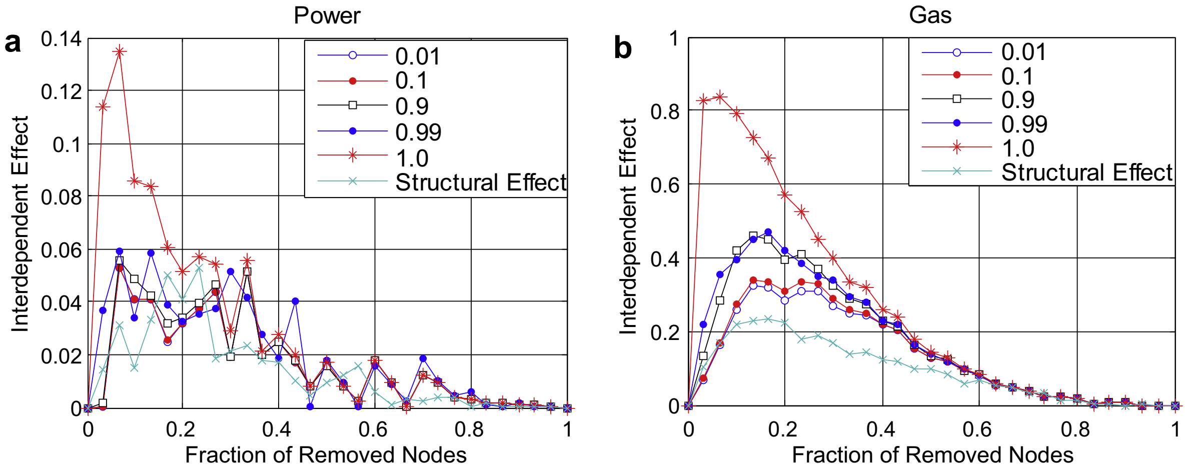

Fig. 8. The interdependent effect in dependence of the fraction of removed nodes for different value of parameter *ps* under random attack. The structural interdependent effect is also shown in the figure. All gas nodes depend on the nearest power nodes to work well while there is only one randomly selected gas-based generator. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator. $tp = 1.5$. (a) Power network; (b) gas network.

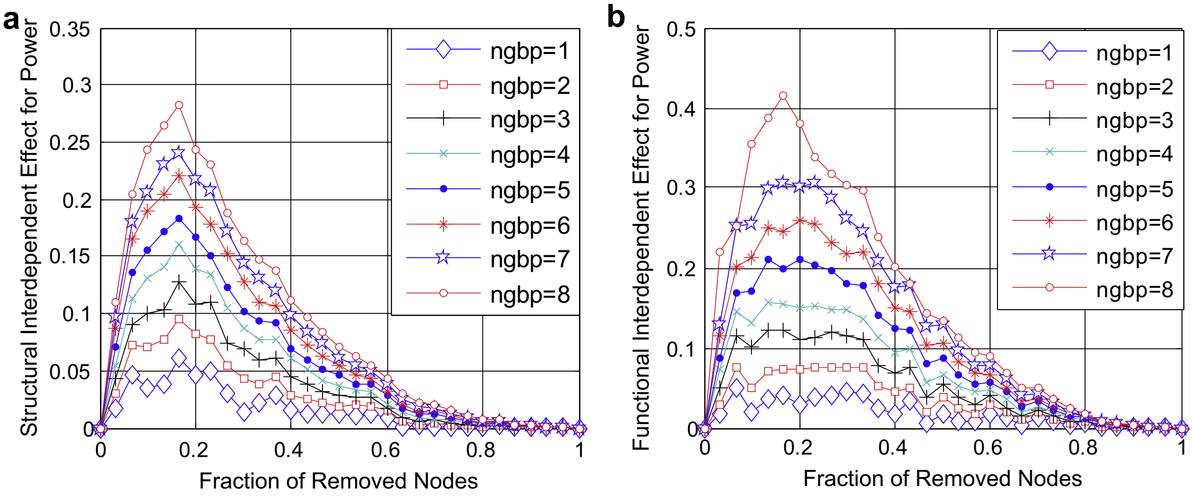

Fig. 9. The structural and functional interdependent effect of gas network on power network in dependence of the fraction of removed nodes for different number of gas-based generators under random attack. All gas nodes depend on the nearest power nodes to work well while some gas-based generators are selected randomly. Simulation results are averaged over 100 runs for random attacks and 20 runs for randomly selected gas-based generator. $tp = 1.5$. (a) Structural interdependent effect; (b) functional interdependent effect.

eters and number of gas-based generators. Results show that no matter it is structural interdependent effect or functional interdependent effect, to cause the largest interdependent effect, their optimum removal fractions are almost the same. These results indicate that analysis on structural vulnerability can offer useful information to understand the functional vulnerability when the investigation on the latter is impossible or difficult. However, to protect our interdependent infrastructures, we should take more consideration on structural vulnerability in the long run while more consideration should be taken for functional vulnerability in the short term.

4. Conclusion

Infrastructures are interconnected and interdependent on multiple levels, their continuous function is essential to support the social and economic organization of productive sectors within a country. The failure of one infrastructure can result in the disruption of other infrastructures, which can cause severe economic disruption and loss of life or failure of services which impede public health and well-being. A methodological approach has been introduced in this paper to comprehensively analyze the vulnerability of interdependent infrastructures. Two types of vulnerability are studied: structural vulnerability and functional vulnerability. For the structural vulnerability, the average reciprocal shortest path lengths of networks are used to measure the detrimental responses when they are subjected to random attacks. For the functional vulnerability, the operating regimes of different infrastructures are further taken into consideration. For different types of vulnerability, our investigations are mainly focused on the interdependent effect. Finally, effect of interdependence strength has been analyzed and relationships between structural vulnerability and functional vulnerability have been also discussed.

The analysis on structural vulnerability will be helpful to design or improve the infrastructures in the long run while the discussion on functional vulnerability will be useful to protect them in the short term. In addition, simulation results show that no matter it is structural interdependent effect or functional interdependent effect, to cause the largest interdependent effect, the optimum removal fractions are both round 18%. These results give some suggestions about strategies for robust design and growth of infrastructures and help the manager to make better decisions on prioritizing scarce resources for mitigation actions.

However, the interdependent infrastructures considered in this paper are artificial, real interdependent infrastructures in a city will be investigated in our future work. And interdependences between two infrastructures are more complicated in reality, for example, some gas pipeline segments maybe have redundant power supply. When a blackout happens, the spare power supply will be used for work. In addition, whether the identical optimum removal fraction for different interdependence strength in interdependent infrastructures can be found from theoretical proof? Whether it holds true for all types of interdependent network structures? All these will be studied in detail in our future direction.

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